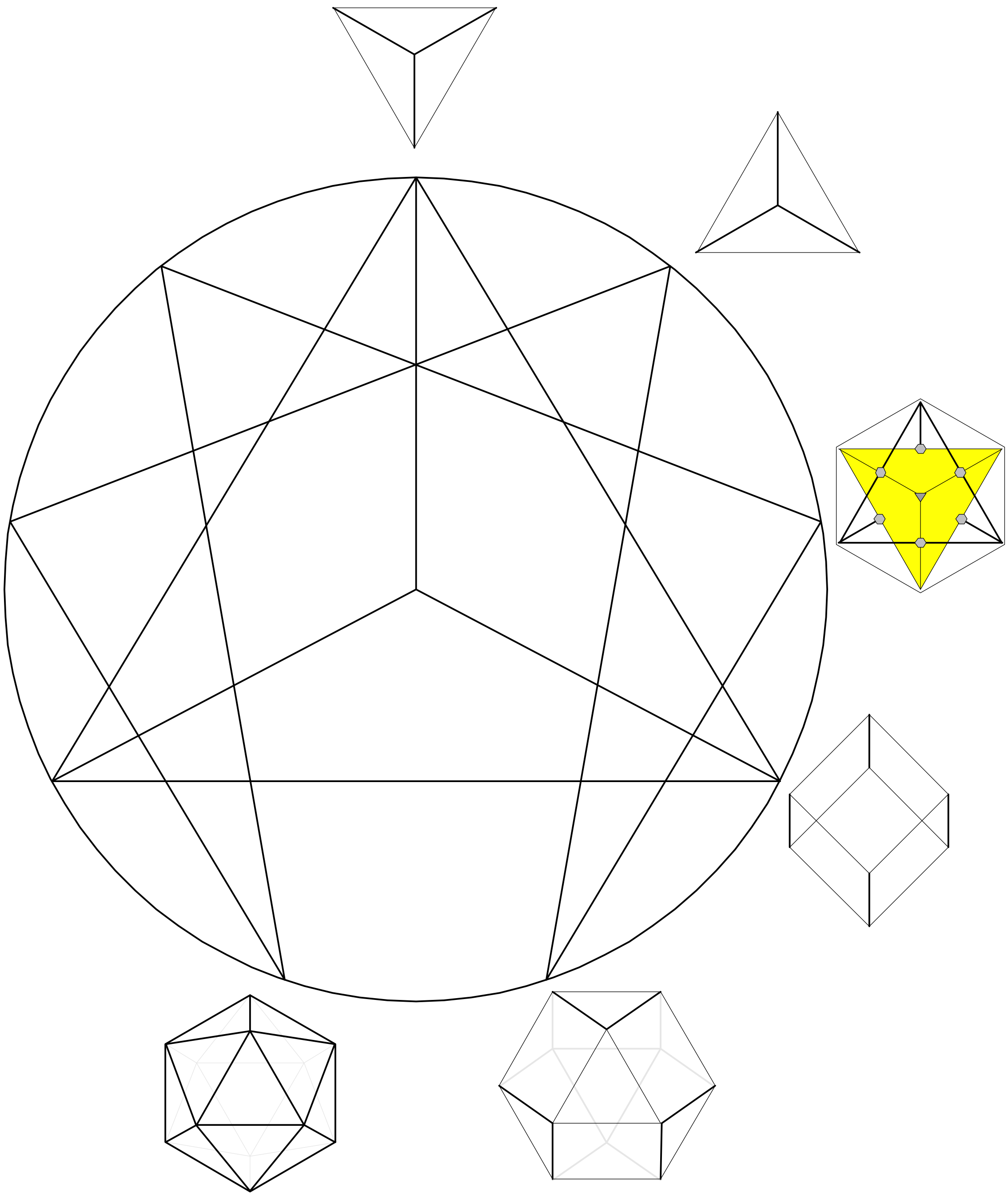
Foundations
of
Number, Structure, & Pattern



Preface

(Edited 1999 July 3)

The theme that unifies this work is one of looking for foundations, of number, structure, and pattern (in space and in time—process and rhythm). I build on foundations set by past searchers for pattern—meditators (zero and infinity), Arabs (zero in base ten), Greeks (Platonic solids), Buckminster Fuller (geodesics, synergetics), John Bennett and those from whom he learned (systematics), Stafford Beer (his varied concepts and Ashby's requisite variety).

These words preface a work that's been in process for well over a year. An inspiration triggered this work. It planted a seed that seems to grow, to bear new fruit, with (at-)tending. The process of producing this work is teaching me, as it comes into clearer focus, *volumes* about the birth, growth, and dissemination of ideas. It has focused my attention and given me a passion to ***express*** that is sorely needed to balance my retentive and generally uncommunicative nature. The words of Gurdjieff and his followers[1] on "the work" describe it as difficult and occasionally painful. Though I can see the truth of this, I also have experienced the joy of discovery that can come from engaging in the process of growth and transformation; that makes the effort *more* than worthwhile. In retrospect, in terms of "the work" and its rewards, my inspiration may have been, in part, fruit from the decades I've pondered a shape that Bucky Fuller dubbed "Vector Equilibrium" (or VE)[2] .

Acknowledgments

I'm indebted first of all to the source that brings inspiration. Also to Tony Blake for his encouragement of and fodder for my mathematical explorations and for events and books he has helped bring into being, to Bucky Fuller for his ideas and as a role-model for dedication to the pursuit of meaning, and to Saul Kuchinsky for including some of my words and pictures in publications of UniS[3].

I couldn't have come to write what's here without the love, support, and ideas of my family, my wife & best friend Bonnie, our soul-friends Stephen & Susan, & others.

I'm expanding and revising this document for 1998, trying to keep it moving towards greater clarity and fuller expression.

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[1] G. I. Gurdjieff lived during the first half of the 20th century. He brought a wisdom to the West—through dance and other physical, mental, and spiritual exercises; his personal way of being; his writings—that has changed the lives of many individuals.

[2] For R. Buckminster Fuller this term relates to a shape with six square and eight triangular sides, whose mass is composed of eight tetrahedrons and six half-octahedrons (pyramids), and whose vertices are in the spatial relation of the centers of closest-packed spheres around an equal-sized central sphere. Until recently, I paid little heed to Bucky's demonstration of the transformation between VE and icosahedron (Platonic solid with twenty triangular sides). That transformation is a key part of the idea that inspired this work.

[3] UniS Institute, P.O. Box 6615, Bridgewater, NJ 08807, USA http://www.infoque.com/unis/

N-Grams (Pattern in Number)
&
Dimensionalities (Structure in Geometric Form)

to

Foundations of
Number,
Structure,
& Pattern

One reason for the change is to make a title that could be the seed for a *Mind Map*, as Tony Buzan (author of many books) uses the term. There's a central word (Foundations) connected to a few key organizing ideas/concepts (Number, Structure, & Pattern).

In *Number*, I discuss zero, a new approach to base 3 (and other odd-number bases, using zero as a central value), and what I call *N-Grams*. I originally wrote the *N-Grams* material (to which I may add the patterns formed in zero-centered odd bases) because I knew that I could from my understanding of the numeric basis for the enneagram, and thought others might come to understand those concepts better through having their structure drawn … and "spelled out." The "spelling out" has given me new insights, too. The *N-Grams* material may also be the basis for some new writing in *Pattern*. In *Structure* is the material from *Dimensionalities*, revised. In *Pattern*, I want to expand on "informing" and "reflecting" patterns. I may also attempt to say something about rhythm as the expression of pattern through time. The various coordinate systems discussed in *Dimensionalities* may have relationships similar to musical notes, and/or which exhibit fractal properties.

There are a few key ideas to me that seem fresh, and worth exploring—ways of considering the topic-at-hand other than the usual.

First is the concept of Vector Measure, particularly as a way of bringing-to-light zero-summing coordinate systems ($2M_1$, $3M_2$, $4M_3$). I think my mathematical intuition is yelling in my deaf ear that this is important, but I can't discount that it may be just an empty ringing. I have yet to explore how angular/polar coordinates might fit with Vector Measures.

Another concept identifies two families of (3D, or M_3) Vector Measures, and how the transition from one family to the other may tie to the 4-5 enneagram transition. I envision this as a step from the mineral world to that of life, of spirit, of potential for transformation.

A third concept is one spurred to paper in response to interest in a "triadic computer."

Number: Zero

Introduction

My mathematical musings tend to include zero in unusual ways.

The way I approach base 3 has zero as the central value.

In what I call N-Grams the top point is special. It tends to fade into the background, yet ties the patterns of all N-Grams together through being part of each. Buckminster Fuller also offers a brief, intriguing idea about this point.

One of the features of some of the Vector Measures I study is that the coordinates add to zero. I have recently learned that these may be what are called Barycentric coordinates.

Zero appeals to my mystical bent as a numeric analogue to the void, *sunyata*, nothingness, the sleep of Brahma. This is one aspect of supreme reality, which also is behind all the being we experience, but in the background, out of focus, not directly visible.

Beyond Zero

The philosophical underpinnings of my exploration are not far from those of Plato and other Greeks, nor from John Bennett's Systematics. The common underlying belief, as I would express it, is that there are certain fundamental patterns from and into which complexity is likely to be formed. From this I surmise that the observable world will often be found to echo these patterns in its structure and process. What I write here is of my attempts to explore portions of the vast domain of mathematics, looking at/for/into fundamentals.

Number: Triadic Base 3

Introduction

Numbers as we know them are a sort of fractal, in the sense that as we change scale, the patterns formed remain similar, if not identical, to those observed at other scales. Two added to three is five, singly or by the millions. A million divided by 7 is 142,857—with one left over to begin the fraction formed from the same pattern of digits[4], repeated for as far as one wants to extend the calculation.

This section contains ovelapping/redundant writings gathered together.

This section describes a measuring stick whose center is zero. The centrality of zero is possible only when using an odd-numbered base. In bases 5, 7, 9, etc. the sides "weigh more" than the center. In base 5, for instance, there are two negative values, two positive values, and zero by itself at the center. With base 3, the "mass" of the center is equal to the "mass" of its left and right (- and +) sides. Each of the three parts (-, 0, and +) holds an equal fraction of the whole.

Bases other than ten

We are accustomed to using and thinking with base ten, though other numeric bases are part of our everyday world as well. We have a dozen and a gross (base twelve), minutes and seconds (base sixty, whether for time or angular measure). The computer's binary (base two) numbers and the derivative octal and hexadecimal have made the concept familiar to others. In any base the number of distinct digits is equal to the base. Base two uses zero and one, base ten uses zero through nine. In numbers with two or more digits, the digits to the left of the initial digit indicate multiples of the base to some power. In base two, two is expressed as 10—one times two (to the first power) plus zero times one. Four is 100, eight is 1000. In any base *x*, the value *x* is expressed as 10—one times *x* plus zero times one. The number 100 represents the base to the second power, 1000 is the base to the third power. Digits to the right of the units digit (separated by a "base point") represent fractions.

Base Two: Binary (base 2) numbers have only two digits, 0 and 1. Left from the "binary point" are the one's, two's, four's, eight's, etc. positions. Going to the right of the "binary point" are the half's, fourth's, eighth's, etc. The binary number 1101.101 represents 8 + 4 + 0 + 1 + 1/2 + 0/4 + 1/8, or 13.625 (1*10 + 3 + 6/10 + 2/100 + 5/1000) in decimal form.

[4] The fraction 1/7 is 0.142857<u>142857</u>…, where the underlining indicates a pattern repeating infinitely.

A triadic base, centered on zero

A variation on the usual way of expressing numbers works only in odd bases, no smaller than three. That is to have digits with negative value, as well as digits with positive value. In base three, the digits would have values one, zero, and minus one. For lack of a better notation, I'll use the symbols [+ 0 -], respectively. Using these digits, there is no need for a minus sign; its purpose is implicit in one of the digits. Numbers in this version of base three have the same one's, three's, and nine's places (three to the power zero, one, and two). Thirteen is expressed +++ (nine plus three plus one). Negative thirteen is expressed ---. Seven is expressed +-+ (nine minus three plus one).

One property of this form of number (with the range of digit values centered on zero) that differs from conventional number systems is that truncating a number is identical to rounding it. If we lop off the fractional part of a decimal number, getting, for instance, 12, the unknown digits might represent a number closer to 13 (as with 12.6 or 12.95) or to 12 (as with 12.1 or 12.495). We have no way of knowing. The same sort of uncertainty occurs at any level of precision.

That doesn't happen with the new scheme. The number +.---... (1 - 1/3 - 1/9 - 1/27 - ...) when extended to any finite number of places will never get as small as 1/2, though one can get as close to 1/2 as one wants by using enough digits. Likewise, the number +.+++... (1 + 1/3 + 1/9 + 1/27 + ...) never gets as large as 1 1/2, though, again, one can get arbitrarily close with enough digits.

A zero-centered, auto-rounding base 3 number system

In the May, 1993 issue of *Dramatic University*, Anthony Blake made brief mention of a triadic computer in his talk (p. 36, first paragraph). I proposed to Tony that a number system such as what I have just discussed might be right for a triadic computer. If a complex calculation were carried out using this number system, each component calculation could be carried out concurrently, feeding into the final result. Components having less effect on the overall result could be calculated to less precision than the more important components. Perhaps nature works this way.

A puzzle solved using a base 3 number system

I applied the above form of base 3 to a math puzzle discussed by Ken Pledge in the May, 1993 issue of *Dramatic University* (what he refers to as a 'reversed notation' on pages 23-25). The puzzle gives you a balance scale and some coins, from which you are to distinguish which coin is counterfeit and whether it is heavy or light. And, you're to use as few weighings with the scale as possible. If the scale balances, you know all the coins on the scale are of the proper weight. If it does not, then either a coin on the heavy side is too heavy, or a coin on the light side is too light. Either could cause the imbalance. As I rephrased the problem: If you have 13 coins, of which no more than one is counterfeit (too heavy or too light), and a 14th coin, known to be authentic, how many weighings will it take to determine if a coin is bad, and if so which, and whether it is heavy or light. There are 27 possible outcomes (all coins OK, coin one to thirteen light, coin one to thirteen heavy).

Three uses of the scale produces 3*3, or 27, results - making it theoretically possible to solve the above problem in three weighings.

To solve the puzzle, start by arranging the digits expressing minus thirteen to plus thirteen in three columns. Each column (1's, 3's, 9's) determines what gets weighed at one time. Each row determines whether a given coin will be weighed, and if so, on which side of the scale. I'll put the rest of the answer in an appendix.

On the Nature of Measurement & the Measurement of Nature— Is Nature not precise ... and redundant?

Precision implies accuracy, but not perfection. I've always been bothered by rounding errors (when I ponder such things). They're endemic to the way we use base 10. Ken Pledge referred me to at least two articles on moving the "zero point" of the digits used nearer to the center. For instance, instead of using 0 to 9, use -4 to +5. None of the articles used odd-numbered numeric bases, and therefore were not truly zero-centered.

• *1 dimension* Let's say you want to find a point on a line. How far is it from "the origin," and how accurate is the measurement? The range covered by 0.5 could be said to go from 0.450…01 to 0.549…9 (rounding) or from 0.50…0 to 0.59…9 (truncation).

Using my base 3 variant, a measuring stick would look like this:

-.			0.			+.		
-.-	-.0	-.+	0.-	0.0	0.+	+.-	+.0	+.+

Redundancy has at least two consequences— you're better prepared if things break, and you're able to catch some errors. When scientists and engineers measure in two or more dimensions, they use the minimum number of axes needed (Cartesian or polar coordinates) — two for a plane, three for a volume, etc. I'm proposing to use more. See the section that follows titled Dimensionalities for more on this.

Appendix

Solution to the problem of weighing the coins

1	0	0	+
2	0	+	-
3	0	+	0
4	0	+	+
5	+	-	-
6	+	-	0
7	+	-	+
8	+	0	-
9	+	0	0
10	+	0	+
11	+	+	-
12	+	+	0
13	+	+	+

To the left is a box with each of the numbers one to thirteen (one row for each coin) expressed in the zero-centered base three. The columns are the nine's, three's, and one's places. Each column determines how to position the coins on the scale—zero for coins not being weighed, and + and – indicating on which side of the scale to put a coin. The fourteenth coin is used as necessary to equalize the number of coins on each side of the scale. The trick here is to use one rule for odd-numbered coins and a different rule for even-numbered coins. For odd-numbered coins let + indicate the right-hand side of the scale and – the left-hand side. Reverse this rule for even-numbered coins.

To convert the results of the three weighings into an answer to the puzzle, one takes the first to have a value of +9 (if the right side is heavier) or –9 (if the left side is heavier). The second weighing would have a value +3 or –3, and the third weighting a value of +1 or –1. Add the results. If zero, no coins are counterfeit. For non-zero results, the absolute value of the answer indicates which coin is off. A negative result indicates a light odd-numbered coin or a heavy even-numbered coin. A positive result indicates the opposite.

Number: N-Grams

Introduction

Following a talk Tony Blake gave some years ago I explored enneagram-like figures in bases other than ten. From that I have developed a graphic generalization that includes the two primary patterns forming the enneagram. These patterns are simply the results of division by an integer in a larger integral base. (The enneagram is composed from the patterns of 1/3 and 1/7 in base ten.) More recently he challenged me with a set of equations {the Blake formulae: [N = A – 1 = a * a] *and* [a + b = A]; where N = number of points around an N-Gram, A = the numeric base, a = informing number, b = reflecting number—I define "informing" and "reflecting" below} that define what I call "second power N-Grams."

The structure of the Enneagram hints at a simple algorithm that defines a way to express graphically the patterns formed through dividing whole numbers. I call the pattern formed from one divisor and a specific numeric base an "atomic N-Gram." Certain pairs of these "atomic N-Grams" form a sequence I call second power N-Grams, a sequence that includes the enneagram.

One way of considering the charts I include is as an instrument panel of gauges to represent numeric patterns. All the gauges up through some large/complex order are implicit in any complex structure/event/entity, but only in certain circumstances will one gauge or another come to the forefront, or be brought into attention/focus/meaning. Simple patterns recur in the more complex patterns. Dyads appear in every other figure in every second row. Triads take two different forms, each appearing every third figure in every third row. N-Gram patterns are mathematical organizing thoughts. Systematics is, in part, an effort towards being conscious of these patterns and towards being capable of working with them in life, or that part of its mix of energy, matter and spirit through time that we experience. Archetypes play a similar role in the unconscious, and dreams. A systematician's approach to number may be similar to the approach of a psychoanalyst or shaman to archetypes.

Consider what comes in the chapter on Dimensionalities as another instrument panel, registering which of various vector systems seem manifest in whatever "system" is being studied. This could be a start at extending systematics past number into geometric structure.

Enneagram of Systematics or of Personality?

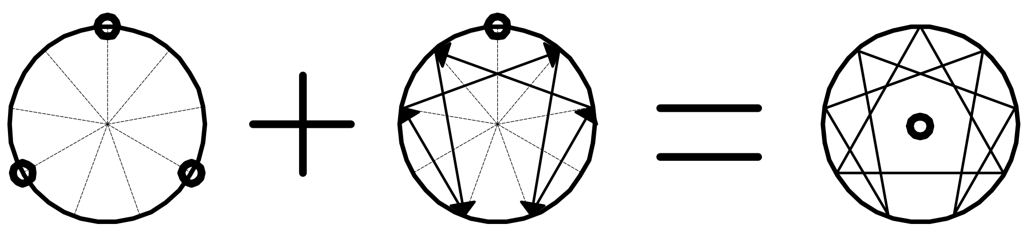
N-Grams would not have been written if the "meme" of the enneagram had not percolated into Western thought. For those who do not know of it, the enneagram is a figure unknown in the West until the 20th century. Its roots seem at least in part through a Mid-Eastern Sufi tradition. Two of its prime introducers into the West were Gurdjieff and Oscar Ichazo.

In the Gurdjieff/Bennett/Systematics tradition, it is an aid in comprehending transformation, in which “higher” and “lower” mix to bring something new into existence. Ichazo’s teaching at the Arica Institute and elsewhere seem to have been the starting point for the ever-growing body of work on the enneagram as a tool for understanding personality[5]. I am working with the enneagram here in its Systematics guise, not as it relates to personality theory. Unlike how it is drawn on the cover of this work, the enneagram’s usual representation does not include the figures around the perimeter nor the three lines drawn between the center and the points of a triangle.

This work seems on *its* way around an enneagram, growing towards a completion still in the process of manifestation. As any work goes around the enneagram’s circle, it can come into new, deeper meaning as manifestation deepens, intensifies, clarifies, aims toward fruition and growing able to handle additional complexity. Whether or not it *does* find new meaning may depend on *requisite variety*—whether the governing aspect of the transformation can connect with “what’s real” in a manner sufficient to some objective.

Construction Details

N-Grams are derived from the mathematical and graphical properties of the Enneagram. Each figure in Charts 1 (by divisor) and 2 (by base) is an ***atomic*** N-Gram, a single pattern (representing division by *x* in base *y*). The enneagram is a ***paired*** N-Gram, made of two atomic N-Grams which each touch the top point (as does every atom) of the circle. Together, the two “N-atoms” (atomic N-Grams) touch every other point on the circle once, and only once. Aside from any esoteric meaning given it, the enneagram is a picture of the



(infinitely repeating) patterns of digits that occur when dividing by 3 and by 7 in base 10.

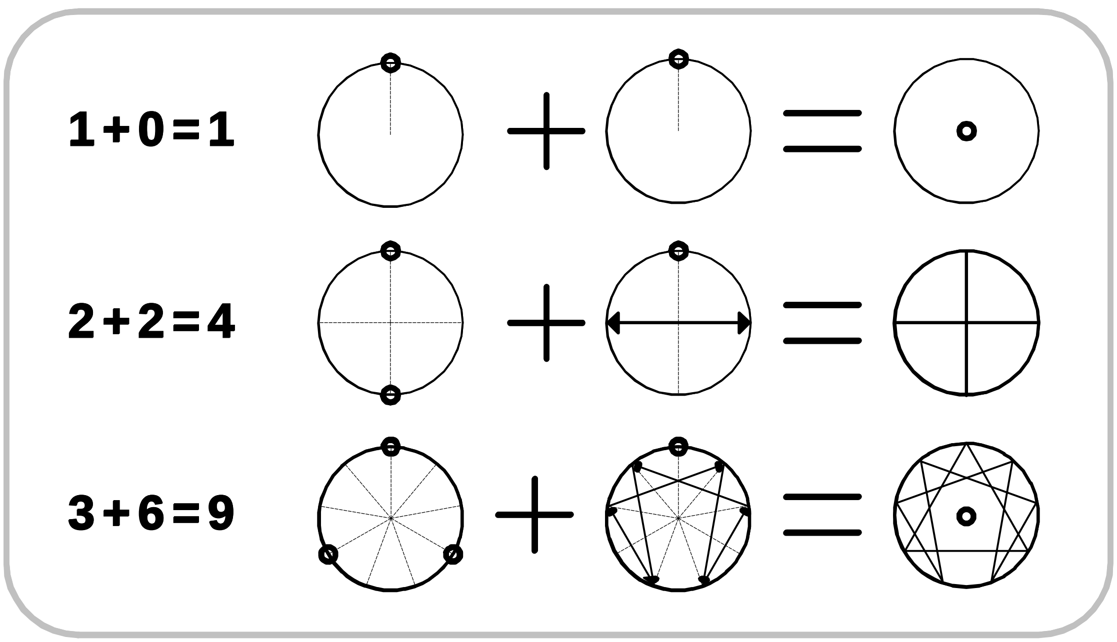
Numbering the points: The figure of the enneagram is enclosed in a circle with nine points around the perimeter, numbered clockwise from 0 (zero) to 9, with *both 0 and 9 at the top point* of the figure. The digits may not be included (zero rarely is) when the enneagram is drawn, but are implicit in any case. To generalize for other bases, the top point in each of the N-Grams represents both 0 and the largest digit (in the numeric base being used).

[5] Only recently have some close to Ichazo spoken and written of their concern with the manner in which the idea of the enneagram was “stolen” and popularized by people who had not gotten much of the essence of what Ichazo taught. Helen Palmer, Riso, numerous Jesuits and others have written perhaps hundreds of books about an enneagram-related personality theory.

[All figures using base *b* have *b*-1 points, numbered (from the top around to the top again) 0 to *b*-1.] Rather than numbering the perimeter points I have used radial dashed lines to mark their position and to distinguish the numeric base of a figure.

Separation of components: The *atomic* N-Grams of Charts 1 and 2 contain a single pattern per figure. Separating the Enneagram into its "atoms" permits including in the pattern for division by 7 a circle at point 9, representing 7/7, in the same way that the circle at point 9 represents 3/3 in the pattern for division by 3.

Progression of Complexity: The numeric and graphic "equations" below display the shapes forming the sequence of paired N-grams at values 1, 2, and 3. They are based on the algorithm mentioned in the introduction, except that the second number is one less than the resolving number (b), as it does not count the shared top point. Note that the number of points around the perimeter is the second power of the number of points in the first figure. These are the ***second power*** N-Grams, discussed in more detail below. In each graphic "equation" the first figure is the **informing** pattern, the second the **resolving** pattern, and the third the paired, second power N-Gram.



Representing repeating digits: The lines inside the enneagram's circle represent two sets of repeating digits. In base 10, 1/7=0.142857142857… where the pattern 142857 repeats indefinitely. The fractions 2/7 through 6/7 are expressed with exactly the same six repeating digits, but with different starting points (2/7=0.285714…; 6/7=0.857142…). Also in base 10, 1/3=0.333…, 2/3=0.666…, 3/3=0.999…. [Mathematically, 0.999… has the same value as 1.000… because the difference between the two can be shown to be less than any arbitrarily small value.] The “classic” enneagram has a triangle connecting points 0369 and a six-line shape connecting points 1428571.

When a repeating pattern has only a single digit (as with 1/3, etc. here), I have drawn a small circle at that point on the N-Gram rather than connecting it by a line to other points. To me this better expresses the nature of the repetition for that point. When a repeating pattern has two or more digits (as with 1/7, etc. here), I have used arrows to connect the points, thereby showing the direction in which the digits form their repetition.

Summary of differences: Compared to the classic enneagram, my atomic N-Gram figures have the following differences.

> Points are not numbered around the perimeter, but radial lines are added to indicate where the perimeter points are, and how many there are.
>
> When repetitions involve a single digit, a circle at that digit’s point is used rather than a line connecting it with other single-digit repetitions.
>
> When repetitions involve more than one digit, arrows are used to indicate the order in which the digits occur.
>
> Each figure represents the pattern formed from division by a single number using a specific numeric base.

Representing patterns that do not repeat: Some divisions form no repeating pattern (other than all zeroes). For instance, 1/2 in base 10 is 0.5. No additional digits are needed to express an exact value. Whenever a divisor creates no repeating patterns, the corresponding circle in the chart is shaded with gray. If a divisor creates some repeating and some non-repeating patterns, the center of its corresponding circle is shaded gray and the repeating patterns that do form are included.

Layout of the figures in the charts: Dividing by a number larger than the base in which the division is being done “over-drives” the calculation. There are not enough distinct digits to “hold” the pattern. Consequently, each row in each of the charts starts with the figures for which base and divisor are the same. [The digits formed in the patterns from “over-driven” calculations may yield some meaning, perhaps setting a tempo for the full expression of the non-overdriven calculation. I’ve yet to explore this tributary/territory. Another unexplored sequence is possible third- or fourth-power N-Grams (2&8, 3&27, 5&125; 2&16, ….]

In Chart 1, each row represents patterns formed from a single divisor. The base in which the calculation is being done increases by one with each new figure across the row. Rows are offset to emphasize the symmetries in the triangle formed by the figures for which the base equals the divisor and the figures for which the base is twice the divisor.

In Chart 2, each row represents patterns formed in a single base. The divisor decreases by one with each new figure across the row. Rows are offset to emphasize the line at the center of all paired N-Grams.

Patterns of patterns

Chart 1: At the edges of the *d*/2*d* triangle: Figures in which the base equals the divisor [b=d] or twice the divisor [b=2d] are gray; they have no repeating pattern. Figures in which the base is one more than the divisor [b=d+1] have a single repeating digit at every point. Figures in which the base is one less than twice the divisor [b=2d-1] contain one (at the top point) or two (at the bottom point for even divisors) single-digit repetitions; all other sub-patterns join pairs of points with horizontal lines (double-headed arrows).

Chart 1: Patterns of gray and prime numbers: Consider the slanted columns starting from the left. In the first, all patterns are gray. Call this column 0. In column 1, no patterns are gray. Column 2 is the first to contain a mix of complete and incomplete figures. Every other figure is incomplete. In column 3, it's every third figure. In column 4 it's back to every other figure being gray. There's a full-gray figure at rows for 2, 4, and 8—the first, second and third power of 2. This is one example of prime factors determining the patterns of part- and full-gray. In column 5 (a prime), it's every fifth figure. In column 6, figures 2, 3, 4 and 6 are gray. The following extends patterns of gray two rows above those in Chart 1.

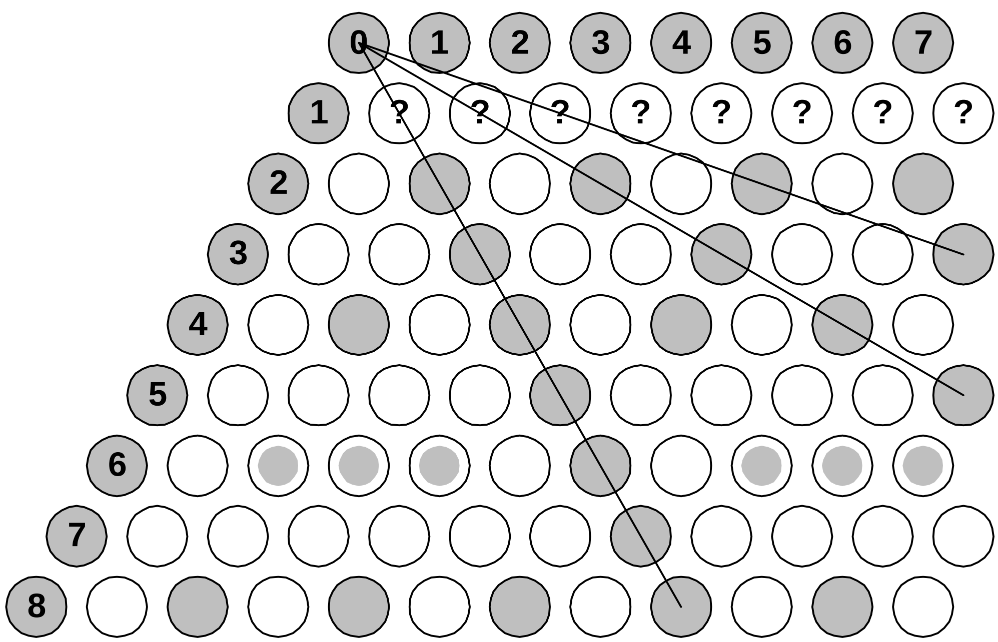


Chart 1: Row 1, gray or not?: The patterns of gray by column indicates that all but the first figure in row 1 should be non-gray. The lines starting at the upper left corner pass through the centers only of gray figures. To be consistent, all the figures in row 1 should be gray. This contradiction is related to 1 being a sort-of but not-quite prime number.

Chart 1: Untouched points: Discounting incomplete (part or all gray) patterns, the number of points left untouched increases by one with each step to the right in any row. In column

1’s patterns, every point has a single repeating digit (a small circle). In column 2 every other figure is incomplete. In each complete figure, the bottom point, halfway around the circle, is the only one not part of a pattern. In column 3, every figure has two free points, at about 1/3 and 2/3 of the way around the circle. Every third figure is incomplete; divisor and base share a factor of 3. The sequence of near-evenly-spaced gaps continues across, with no gap between the top point and those adjacent to it until the divisor is less than half the base.

Chart 1: Repeating sequences of patterns: A set of unique patterns is formed between the gray circles at the figures in which the base is equal to the divisor and to twice the divisor. The same set of patterns repeats between bases two and three times the divisor. Where there was no gap in the first pattern, there is now a gap of one untouched point, and where there was a gap of one untouched point, there is now a gap of two. One more untouched point is added to each gap in the set of patterns between three and four times the divisor.
[This sequence seems to hold “forever,” though I don’t have a mathematical proof.]

Chart 2: Paired N-Grams: In the second chart a vertical line marks a sort of mirror. Non-gray figures equidistant right and left of the line form paired N-Grams.

Primes: Whenever the base or the divisor is a prime number, there are almost no incomplete patterns. They occur only for N-Grams for which the base is an integral multiple of (1, 2, 3, etc. times) the divisor.

Dis-counting complexity: There are other aspects to these figures for which my thoughts are not yet well formed. One aspect has to do with needing a system larger than that being observed to “see”/record/model what is there/going on (matter, pattern/events through time).

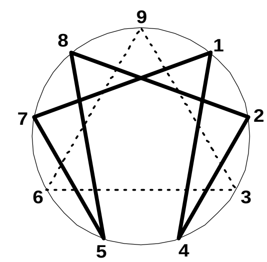
The number of points around the circle must be one less than the base for them to represent points of repetition (.111… rather than .100…).

The base must be larger than the divisor for patterns to appear (without the confusion that occurs when the base is less than the divisor).

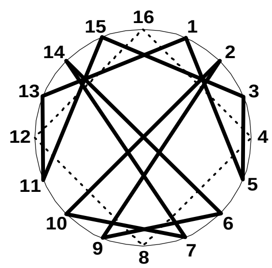
The patterns that occur when the base is one more than the divisor are so regular as to be “uninteresting.” One needs to go to a base larger by at least two to get sub-patterns that touch two or more points. In those, the top-most point always “sits by itself,” thereby becoming less visible, “disappearing” to some extent. [To get a sub-pattern containing n points one must divide by at least n+1 in a base of at least n+3.]

The time it would take me to examine third (1, 8, 27, 64, 125, …), fourth (1, 16, 81, 256, 625, …) and higher powered N-Grams has kept me from the task. (It takes a while to map out, for example, what patterns 1/5 and 1/121 would form in base 126.)

Second-Power N-Grams



Tony Blake presented to me the challenge of looking for pattern in a sequence of figures that includes the enneagram (*figure at right*) and a sixteen-pointed figure (*below, left*). The number of points around the perimeter of each figure in this sequence is the second power of an integer. The enneagram has nine (three times three) points.



I call three its *informing* (or: *generating, implicit*) *number*. Only the informing number and an algorithm implicit in the enneagram are necessary to create one of the figures. The figure to the left has an informing number of four. The algorithm that creates these can be described using graphic as well as numeric steps.

The first step sets the number of points around the perimeter of the figure to be the second power of the informing number. The next step expresses the informing number as a shape or pattern, the *informing pattern*. It includes the top point of the figure, and each n^{th} point of the n^2 points around the circle. (For example, the informing number three in the nine-pointed enneagram creates the triangular informing pattern touching every third point around the circle—3, 6, and 9.)

The remaining n^2–n points (and the top point again) are contained in a set of (sets of) *resolving* (or: *complementary, reflecting, explicit*) patterns. As the informing number increases, the (sets of) resolving patterns manifest in a sequence of increasing complexity. The resolving patterns (each, other than for the top-most point, is more than one digit) are dynamic, as compared to the static informing pattern, in sort of a Shiva/Shakti relationship. That is, the informing pattern contains a seed, but is static, not yet fully engaged in the process of manifestation. The resolving patterns dance around the static points of the informing pattern in the complexity and aliveness of active being.

Mathematical aside: (1): For informing number n, the informing pattern and its complement are defined by sequences of repeating digits generated by division in a numeric base equal to n^2+1. Division by the informing number defines the informing pattern. Division by n^2+1-n (the numeric base minus the informing number) defines the complementary patterns. The triangle in the enneagram comes from the patterns of repeating digits (0.333…, 0.666…, 0.999…) expressed in division by three in base ten. Its complement comes from division by seven (the base, ten, minus three, the informing number).

Every pattern, informing and resolving, is symmetric about a vertical axis. That is, its left and right sides are mirror images.

The full set of resolving patterns for an informing number is composed of one or more series of individual patterns. Each series starts with one pattern of length 2, then 6, then two patterns of 6, then three, four, and so on, adding one more pattern of 6 each time. Every third figure (with informing numbers 2, 5, 8, …) starts a new pattern. The table below contains on each row the values for the points touched by one of the complementary patterns in a given figure.

For every third informing number a new pattern of length 2 starts a new series which will grow (to 6, two 6’s, three 6’s, …) with the informing number. In the first series, pattern p (p from 1 to n-2) starts at point p, jumps to p past the p^{th} point in the informing pattern, then jumps to one before the p^{th} point in the informing pattern. Then it jumps to the first of the three points in the mirror image of the pattern just described, after which it jumps back to the starting point. In the enneagram, the pattern goes from 1 to 4 (1 past 3) to 2 (1 before 3), then to 8, 5, and 7 (the mirror image of 1, 4, 2) after which it goes back to 1. The second pattern in the figure with informing number 4 starts at point 2, goes to 10 (2 past 8), then to 7 (1 before 8), then to 14, 6, and 9 (the mirror image of 2, 10, 8) and back to 2.

Resolving Patterns (lengths 2 & 6) in Second-Power N-Grams # 2-10

Fraction	Repeating Digits					
Informing Number = 2						
1/3	1	3				
Informing Number = 3						
1/7	1	4	2	8	5	7
Informing Number = 4						
1/13	1	5	3	15	11	13
2/13	2	10	7	14	6	9
Informing Number = 5						
1/21	1	6	4	24	19	21
2/21	2	12	9	23	13	16
3/21	3	18	14	22	7	11
1/3=7/21	8	17				
Informing Number = 6						
1/31	1	7	5	35	29	31
2/31	2	14	11	34	22	25
3/31	3	21	17	33	15	19
4/31	4	28	23	32	8	13
8/31	9	20	10	27	16	26
Informing Number = 7						
1/43	1	8	6	48	41	43
2/43	2	16	13	47	33	36
3/43	3	24	20	46	25	29
4/43	4	32	27	45	17	22
5/43	5	40	34	44	9	15
9/43	10	23	12	39	26	37
10/43	11	31	19	38	18	30

Fraction	Repeating Digits					
Informing Number = 8						
1/57	1	9	7	63	55	57
2/57	2	18	15	62	46	49
3/57	3	27	23	61	37	41
4/57	4	36	31	60	28	33
5/57	5	45	39	59	19	25
6/57	6	54	47	58	10	17
10/57	11	26	14	53	38	50
11/57	12	35	22	52	29	42
12/57	13	44	30	51	20	34
1/3=19/57	21	43				
Informing Number = 9						
1/73	1	10	8	80	71	73
2/73	2	20	17	79	61	64
3/73	3	30	26	78	51	55
4/73	4	40	35	77	41	46
5/73	5	50	44	76	31	37
6/73	6	60	53	75	21	28
7/73	7	70	62	74	11	19
11/73	12	29	16	69	52	65
12/73	13	39	25	68	42	56
13/73	14	49	34	67	32	47
14/73	15	59	43	66	22	38
21/73	23	48	24	58	33	57

Fraction	Repeating Digits					
Informing Number = 10						
1/91	1	11	9	99	89	91
2/91	2	22	19	98	78	81
3/91	3	33	29	97	67	71
4/91	4	44	39	96	56	61
5/91	5	55	49	95	45	51
6/91	6	66	59	94	34	41
7/91	7	77	69	93	23	31
8/91	8	88	79	92	12	21
12/91	13	32	18	87	68	82
13/91	14	43	28	86	57	72
14/91	15	54	38	85	46	62
15/91	16	65	48	84	35	52
16/91	17	76	58	83	24	42
23/91	25	53	27	75	47	73
24/91	26	64	37	74	36	63

In an appendix I include algebraic descriptions for more of the patterns in the various series.

Structure: Dimensionalities

Introduction

I've read the tale of R. Buckminster Fuller's first exposure to structure many times. Bucky could never see well. In Kindergarten (I think), he and his classmates were given peas and toothpicks with which to construct more complex objects. The rest of the class made cubes, such as they saw in the world around them. Bucky formed tetrahedrons, the shape itself informing his fingers of an inherent stability not found in cubes. From this seed he began his life-long explorations of structure.

Picture yourself sitting somewhere, some-when, checking out your stability in the surrounding environment, your connections to the things or entities around you. Are you part of a larger whole? Are you a distinct entity relating, perhaps, to other entities? Perhaps you're an atom, a molecule, a star; perhaps a corporation, even a species or a biosphere… surrounded by some small or large number of external objects/systems with which you (the system-in-focus) interact to varying degrees…

1. Thank you, R. Buckminster Fuller

Three of the Platonic solids (tetrahedron, cube, octahedron), plus Bucky Fuller's Vector Equilibrium (VE—8 triangular & 6 square faces) all contain systems of 3, 4, and 6 vector-measure (VM). The remaining two (of 5 total) Platonic solids (dodecahedron, icosahedron) each contain systems of 6, 10 and 15 vector-measure (VM).

2. Thank you, creative sparks

I picture one family of vector systems ($3M_{3,}$ $4M_{3,}$ 6_0M_3 and 6_1M_3) on the right side of the enneagram (points 1 to 4) which manifest through space homogeneously. No one spot is any more a center point than any other. On the left (points 5 to 9) are vector systems ($6_2M_{3,}$ $10M_{3,}$ and $15M_3$) that imply at least the potential for self-contained entities distinct from their surroundings. The vector systems discussed in this paragraph work in 3Space (M_3)—that which we define and measure using 3 *Des*-Cartesian coordinates (x, y and z).

4. Thank you, Tony Blake, JGB, …

Simple division produces patterns of digits. These patterns vary as one changes the base of the number system being considered. The most complex of the patterns implicit in base 10 are expressed in the enneagram. Simpler and more complex patterns can be found in bases under and over 10.

Enneagram with VE and Icosahedron at points 4 and 5

This work was sparked when I sensed how one transition in the enneagram (between points 4 and 5) might connect to the transformation between VE and icosahedron (that Bucky so often described, and related to 6_1M_3 and 6_2M_3 in my terminology). This clarified for me the two "families" of structural components found in the five Platonic solids (tetrahedron, octahedron, cube, dodecahedron & icosahedron) and a few related shapes (VE among them).

Having the transition occur at the jump from 4 to 5 seems quite appropriate. The number 5 clearly appears in the structures of the icosahedron and dodecahedron, but not in the tetrahedron, octahedron, cube, and VE that I conceptualize as being associated with points 1 to 4 of the enneagram. Five is the number of triangles around each vertex of the icosa— (I'll now shorten "hedron" to "—"). Five is also the number of edges of each face of the dodeca—, and is necessary to calculate the "golden ratio" first found manifest in the more complex shapes. This same "golden ratio" is expressed in the patterns of growth of flowers and other forms of life.

M_0 M_1, M_2 M_3

M_0: What *is* a "null point"-a point from which nothing is measured?

M_1: A line, a vector, a path through space, diverted on occasion by forces from larger systems.

M_2: A plane, a surface, a "field of play."

M_3: 3D-the Cartesian world of our scientists & engineers.

If we listen to Newton, and wish precision
in our measurment of the world manifest around us
all the bits of reality we can touch with one or more senses,
all of that which is a reflection of the World that lies behind—

then we must "measure twice, cut once." That is, we recognize that any measurment we can make will always, fundamentally, be an approximation.

Newton

In Newton's words (as per *The Intelligent Enneagram*, Blake p. 93), his three laws are:

(1) "Every body continues in its state of rest, or uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it."

(2) "The change of motion is proportional to the motive power impressed and is made in the direction of the right line in which that force is impressed."

(3) "To every action there is always opposed an equal reaction, or the mutual actions of two bodies upon each other are always equal and directed to contrary parts."

Blake's (self-proclaimed crude; *op.cit.* p. 82) paraphrasings are:

(1) "Things keep going the way they are until something else disturbs them."

(2) "The amount of disturbance is dependent on the energy brought in from outside."

(3) "Action and reaction are equal and opposite, and the whole situation remains the same as before."

I think the third law is a special, boundary case applying to 1Space (M_1-that which we define and measure using 1 Cartesian coordinate). More generally it might be stated:

(3) The sum of all actions (amongst components of the "system-in-focus") is zero.

We can only ever approximate when we attempt to measure the "real world" (a distance, area, volume, weight, luminance, pH, etc.), to describe (define and measure) a dimension, along a VM. Newton's third law implies an observation along one dimension (*i.e.*, in M_1) in which 2 measurable events occur—an action and a reaction. We (unthinkingly) take one measure as sufficient. If we were "gods"—if we could obtain a perfect measure—this would suffice. As we are not, redundant measure will give us a sense of the minimum error in our measures (the degree to which our measures disagree) as compared to the accuracy with which we are attempting to measure (the nearest parsec, mile, inch, micron, angstrom). That is, how much "play" (inherent in the mechanics we use to make our measurement—see my writings on a triadic number system) do we expect in our best approximations?

The re-phrased third law applies only to $2M_1$, $3M_2$, $4M_2$ (M_2 where each of two $2M_1$'s add to zero), $4M_3$ and to 6_0M_3 (M_3 where each of three $2M_1$'s add to zero). Other statements are needed to express the more complex zero-ings that can be found in the other VMs.

Terminology of: [M_0 M_1, M_2 M_3], [n_xM_x]

Comparing Dimensionalities

The simplest relation to the exterior (that which is not-us) is one in which there are *no* objects (M_0 in the terminology I'll use). As soon as an object appears, when the environment comes into consciousness, one must consider whether one is measuring in M_1, M_2, M_3 or M_x space. In M_1 space there is distance, but direction is only implicit, unless/until one establishes a framework, up & down, right & left, in front & behind. In M_2 space, *three* establishes a new degree/kind of stability. $2M_1$ to $4M_2$ is a "mechanical" extension, whereas $2M_1$ to $3M_2$ is more spare, as well as more stable. The transition from $2M_1$ to $3M_2$ has precisely the same "surprise" feel to it as does $3M_2$ to $4M_3$—a "surprise" new dimension, affecting all "already-in-focus" dimensions in some semi-orthogonal way.$_1$ The last sentence might be better understood with a physical image. The transition from $1M_0$ to $2M_1$ could be visualized as pinching and pulling out two strands from an always-central rubber ball. $2M_1$ to $3M_2$ would be pulling out a third strand, the ends of the strands forming a triangle. $3M_2$ to $4M_3$ would be pulling out a fourth strand, forming a tetrahedron.

So we have at least three kinds of extension from one M into an M of higher dimension—

- a vector (or set of vectors) independent/orthogonal to all existing vector-measures
- a vector "semi-orthogonal" to all existing vectors, one that distorts them (maximally efficient?) so that all vectors in the new system look the same *and* sum to zero
- a vector (or set of vectors) not orthogonal (though with other interrelationships, such as when going from $3M_2$ to $6M_3$) to the existing set of vector-measures

One of Jack Seltzer's reactions to this work was to talk about wave versus particle. I wonder whether measures XM_Y (where x>y) need to be used to describe waves, and their interactions. I also wonder of connections to quantum phenomena.

What's constant across different VM's? That which is being measured. What's being measured in M_1 is less complex than what's measured in M_2.What's in M_2 is less complex than what's in M_3. There are interrelations between M_1 and M_2, as there are in a more complex fashion between M_2 and M_3. etc. (and such an etc.!)

In M_1, we take one measure. Netwon says two. These are $1M_1$ and $2M_1$.

In M_2, we can take two measures (Descartes)—$2M_2$, four (Newton&Descartes)—$4M_2$, or three (Newton extended)—$3M_2$.

In M_3, things get lots more complex.

$3M_3$—6_0M_3

Applying, then, $2M_1$ along each vector/ axis (-x, +x), and extending to M_3, we use 6 measures (-x, +x), (-y, +y), (-z, +z) which add to zero in each pair, and overall. This is a relatively "mechanical" use of 6 dimensions to measure M_3, termed 6_0M_3.

6_1M_3

Here, the 6 dimensions are not in 3 pairs, but in 4 orderings, each ordering consisting of 3 vectors in the form $3M_2$ defining a plane cutting M_3 "in half" (and forming a great circle), and the other 3 vectors clustered around the perpendicular to that plane. The perpendiculars to the 4 planes are the 4 vectors of $4M_3$.

$3M_3$, $4M_3$, 6_1M_3

$3M_3$ is our "normal, everyday" Cartesian system, one of a minimum number of measures. In it and in 6_0M_3 the vectors can be visualized as coming from the center of a cube to the center of each of its 6 faces.

$4M_3$ is composed of 4 vectors, which can be visualized as being from the center to the vertices of a tetrahedron (or to the centers of its faces), and which always sum to zero.

6_1M_3 is composed of vectors each (of which can be visualized as being) through the center and through one of the 6 pairs of opposing vertices of Fuller's VE.

VMs and Platonic Solids

The Vector Measures $3M_3$, $4M_3$ and 6_1M_3 are all implicit/contained in the tetrahedron, octahedron, cube and VE. In all cases, the vectors can be seen a going through (or emanating from) a central point.

The cube (hexahedron) is probably the most familiar of the Platonic solids. $3M_3$ is formed by vectors through the centers of the cube's 3 pairs of opposing faces. One direction (center to face) is chosen as "positive," making the vector in the opposite direction "negative." $4M_3$ is formed by vectors through the 4 pairs of opposing vertices (corners) of the cube. 6_1M_3 is formed by vectors through the centers of the 6 pairs of edges of the cube *or* the octahedron.

In the tetrahedron, $3M_3$ is formed by vectors through the centers of 3 pairs of opposing edges (which, when looked at along the vector in question, form a cross—lines at right angles). $4M_3$ is formed by vectors through the 4 vertices and their 4 opposing faces. The tetrahedron is the only Platonic solid with vertices opposite faces. 6_1M_3 is formed by vectors parallel to the 6 edges of the tetrahedron.

In the octahedron, $3M_3$ is formed by vectors through the 3 pairs of vertices opposite each other. $4M_3$ is formed by vectors through the 4 pairs of opposing faces. 6_1M_3 is formed by vectors through the 6 pairs of edges opposite each other.

The VE's 6 pairs of vertices opposite each other form the vectors of 6_1M_3. $3M_3$ is formed by vectors through the center of the VE's 6 square faces. $4M_3$ is formed by vectors through the center of the VE's 8 triangular faces.

6_1M_3—6_2M_3 VMs (4, 5)

There is a "shock" going from 6_1M_3 to 6_2M_3. Each of the 6 squares on the surface of the VE is sub-divided into a pair of triangles by joining one of the two pairs of opposing corners of the square. The VE can "collapse" to form either of two icosahedrons (the diagonal pulled across any one square face determines the diagonal on each of the other 5 square faces).

6_2M_3, $10M_3$, $15\ M_3$

6_2M_3 is defined by vectors through the 6 pairs of opposing vertices of an icosahedron. $10M_3$ is defined by vectors through the 10 pairs of opposing faces of an icosahedron. $15M_3$ is defined by vectors through the 15 pairs of opposing edges of an icosahedron.

6_2M_3, $10M_3$, and $15M_3$ are also implicit in the dodecahedron, in a similar fashion as above.

There is another figure, for which I do not have a name, that has the same relationship to the icosahedron and dodecahedron (formed by all the mid-edge points, where nested figures touch) as the VE has to the cube and octahedron. It's also the relationship the octahedron has to a pair of nested tetrahedrons. It has 60 vertices; 60 edges; and 20 triangular and 12 pentagonal faces. The 60 vertices define a seventh VM, $30M_3$.

Some Thoughts

While in the Vermont woods, transforming bits of the landscape, including a little weaving of branches, the concept of what minimum structure to create rope—3 in a braid, or 2 twisted pairs? Either will maintain its integrity. These are related to measures ($3M_2$ and $4M_2$) in M_2 space. Holding the rope's strands together requires forces perpendicular to the rope's length.

Where do the bits being juggled fall, naturally, on the enneagram? Are there multiple patterns that make sense; that help explain how pieces fit together? Tony Blake has suggested transposing the inner 1-4-2-8-5-7 and outer 1-2-4-5-7-8 patterns as a general method of looking for new connections.

Bucky tends to perseverate on the great spheres; I on their perpendicular vectors. This might have something to do with wave *vs*. particle, too.

I write this revision in late July 1997. Last December I conceptualized a pair of enneagrams, one beginning as the other leaves off. The triangle points on the first (from top to top) are 0, 1, 2, 3; and on the second are 3, 4, 5, 6. The second enneagram is much like the picture on the cover of this paper. At the bottom of its (3-6-9) triangle are a cube and a dodeca—, one with square (4-edged) faces, the other with pentagonal (5-edged) faces. In both cases each vertex is the corner of 3 of the solid's faces. A figure with triangular (3-edged) faces falls naturally at the "0" point at the top (or point 3 in the double-enneagram sequence), a tetra— which becomes a dual to the tetra— at point 1 on the second enneagram. The "9" point at the top becomes a paradoxical figure, a solid surfaced with hexagons.

Structure and Pattern

Structure and pattern are found in number (patterns of whole numbers and approximations to a standard of preciseness) and shape (e.g., the Platonic solids).

N-Grams, various 2D & 3D figures, and numbers (*yes*, like 0, 1, 2, 3; *exactly* like that) all work in simple relationships. When combined, they *may/can* increase the level of complexity that can be described (*requisite variety*—thanks to Stafford Beer for introducing me to this concept) as more relationships are considered simultaneously.

As Complexity Increases, It Complexifies Pattern

The enneagram points out how things metamorphose as they grow more complex. For example, consider the sequences 1,4,2 and 8,5,7,1—and their combination.

The first advances across the boundary 1—2, the gap between two numbers such as also occurs at 4—5 and 7—8. All these are transitions from one triad to another. For instance, the boundary 1—2 is between the triads centered at 9 and 3. The second sequence (8,5,7,1) advances across a more significant boundary, 8—9—1, containing transitions of completion and initiation. The two sequences together form a meta-pattern, a sequence of sequences, which takes one across the transition achieved by cycling through a complete enneagram.

The first has a single step between first and last step; the second has two steps in-between them. Together, there's a higher level transformation going all the way around the loop, 9—3—6—9; two steps between beginning and end-which mark the same spot, but completely different because something has now gone fully once around the circle.

Part-way Round the Enneagram through Solids

There's a structure to the sequence of solids as well as to the dimensionalities contained within them. Tetra— starts M_3, then a pair of tetra—s form an octahedron. Next, take each vertex of the octa— as the center of a square surface; *et voilá!* A Cube! Cube and octa—nest; their edges cross in the same way two tetra—s join to form an octa—, forming Bucky's VE (Vector Equilibrium).

Keeping points on the surface identical, pulling a diagonal across all 6 squares on the surface (in either of two possible patterns) forms an icosahedron. As Bucky points out, this is equivalent to pulling the sphere from the center of 12 of identical size surrounding it, or shrinking the one at the center. This transition really seems to fit the 4-5 transition in an enneagram, into a world where 5 exists in the solids' characteristics and ratios.

Next comes dodecahedron, nest-mate of icosa—, together forming a seventh structure with 15 pairs of vertices. I don't know *what* transforms will take it to something at point 8 on the enneagram. Point 9 may be the hexagonally—surfaced solid mentioned earlier.

N-grams

In the patterns of the N-grams, 0 and the highest digit overlap at the "top" of the N-gram. In base 2, the only pattern formed by division (by 1) is the repeating decimal, 0.111…. The "difference" between 0.111… and 1.000… seems to have something to do with that "higher energy” that needs to come from the state of completion to bump the rising energy through the “top” of the N-gram into the start of a new cycle.

baraka and higher systems

What is it that would come from the “higher system” which simultaneously recognizes $3M_3$, $4M_3$, 6_1M_3, 6_2M_3, $10M_3$, and $15M_3$? What is it that would take us to completion of the enneagram of dimensionalities?

Appendix

To express the patterns mathematically, I'll use i for the informing number, and j for the index in each series of patterns. If i mod 3 = 2 (*i.e.*, i=2,5,8,…) then the first pattern, of two digits, has $(i^2-1)/3$ and $(2i^2+1)/3$.

As the two digits are mirror images (symmetric around a vertical axis), they add to i^2. Likewise, all patterns of six digits contain three pairs of points (e.g., 1&8, 4&5, 2&7) that add to i^2. The last three digits are i^2 minus the first three digits (the first and fourth, second and fifth, third and sixth digits each add to i^2). The first series of patterns of six digits starts at i=3. For values of i >=3 there are i-2 patterns in the first series. The first three digits in the series are j, $(i+1)j$, and $ij-1$. The second series starts at i=6. For values of i >=6 there are i-5 patterns in the series. The first three digits in the series are $i+j+2$, $(i+1)(j+2)-1$, and $i(j+1)-2$. The third series starts at i=9. For values of i >=9 there are i-8 patterns in the series. The first three digits in the series are $2i+j+4$, $(i+1)(j+4)-2$, and $i(j+2)-3$. I haven't worked out the series beyond this point, but it would seem they would follow an equally simple pattern of digits.

The follwing table expresses the underlying "algorithm" of the second-power N-Grams. See the earlier table of second-power N-Gram patterns to extend this past informing number 4.

Informing Number	Informing Pattern	Reflecting Pattern
1	[1]	[1]
2	[2], [4]	[4], [1 3]
3	[3], [6], [9]	[9], [1 4 2 8 5 7]
4	[4], [8], [12], [16]	[16], [1 5 3 15 11 13], [2 10 7 14 6 9]

